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Teaching Mathematics Excursions Using Umbrella Themes

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Math Excursions is a type of course taught at many colleges across the U.S. It has also been called "Math for Poets", "Liberal Arts Math", "Math Appreciation" and the like. The idea is to show students what mathematicians really do and why they just *love* doing it. Teachers attempt to introduce their students to the beauty, applicability and pervasiveness of Math and to remind them of its fascination. It's not a survey course in the old sense of covering many topics in very little depth, but rather it might consider 3 or 4 topics with sufficient opportunity for exploration to achieve the goals listed above. In this essay, I describe the concept of the course, briefly touch on some of its folk-history, offer thanks for its existence and suggest using an umbrella theme as a good way to teach it.

Starting in the 1930's, several books have been published which I believe laid the groundwork for the present conceptions of a Math Excursions course. Some of these are books for the layman with no exercises, and some are college textbooks. In 1930, "Number, The Language of Science" by Tobias Dantzig appeared; it was directed toward the mathematically untrained but intelligent layperson and dealt with the story behind the dry techniques that were usually taught as being all there was to mathematics. Einstein found this revolutionary little book to be "beyond doubt the most interesting book on the evolution of mathematics that has ever fallen into my hands." In his preface, Dantzig describes his effort as pioneering and it was just that-- it is what we would today call a work of 'Humanistic Mathematics.' He started by describing the number sense in animals and mathematics' pre-history and early history, and then dealt with number mysticism, introduced the people who created mathematics and discussed the role of intuition in the creative process. Fundamental issues were raised without bringing in their whole intricate mathematical apparatus-- it was a book mainly of ideas not methods. Many of the mathematics topics Dantzig wrote about were new to the popular audience: infinity, number theory, the ideas of calculus, Galois theory, other bases for numeration, etc. In content and approach, this little book has always struck me as presenting the blueprint for the modern Excursions course.

Other works in a similar or complementing spirit appeared soon thereafter: E.T. Bell wrote “Queen of the Sciences” and “Men of Mathematics” in the 30’s, “Mathematics for the Million” by Lancelot Hogben also appeared in the 30’s and “Mathematics and the Imagination” by Edward Kasner and James Newman in the 40’s. In England W.W. Sawyer’s “Mathematician’s Delight” became available in the 40’s too. At a higher level, “What Is Mathematics” by Courant and Robbins (another Math book that Einstein praised), was published in 1941; it was the first book with exercises to appear that I know of that recognized that there was a problem with the way Math textbooks were written and courses taught, both for majors and liberal arts students. This from the Preface: “The teaching of mathematics has sometimes degenerated into empty drill...[that] does not lead to real understanding or greater intellectual independence...The goal is genuine comprehension of mathematics as an organic whole...”

What an avalanche these works have since engendered! In recent years the great number of popular math books published has been astonishing-- consider the works of John Paulos, Simon Singh, Martin Gardner, Theoni Pappas, Ian Stewart, A.K. Dewdney, Ivars Peterson, Rudy Rucker, William Dunham, Paul Hoffman, Sherman Stein and Keith Devlin for starters. These are serious books written for the well-educated and curious layperson on a variety of novel excursion-type subjects. But they are not text books-- they have no exercises, they are not required, they are not expensive. Often in paperback, they are written to entice, amuse, and share a mathematician’s love for his/her art with the serious reader.

In 1956, Kemeny, Snell and Thompson of Dartmouth College, explained in the preface of their textbook, “Introduction to Finite Mathematics”, that college freshmen need a course that introduces them to “concepts of modern mathematics ...to include applications to the biological and social sciences and thus provide a point of view, other than that given by physics, concerning the possible uses of mathematics.” Wishing to enrich their students’ early mathematical diets beyond pre-calculus and calculus, their aim was to chose topics “close to the students’ experience, which are important in modern day mathematics, and which have interesting and important applications.” They felt this would best be done by restricting themselves to “finite problems...which do not involve infinite sets, limiting processes, continuity, etc.” They chose logic, sets, counting, probability, vectors and matrices, linear programming, the theory of games, and included many applications to the behavioral sciences. Thus, a new packaging of mathematics topics, the Finite Math course was born, a course that owed something to Dantzig, Kasner, Newman and similar authors in that it wanted to show students and readers some of the heretofore advanced delights that only mathematicians played with, that math was more than dry technique, and more than pre-calculus and calculus. While it eschewed many of the Dantzig delights such as infinity, number theory, and an emphasis on cultural content, what it brought to the table was discrete mathematics topics, interesting, novel and modern non-physics applications and, of course, that it was a textbook with exercises for use in a college course.

Further advances towards the present-day college introductory liberal arts mathematics course were made by three groundbreaking texts published during the decade following Kemeny's book-- "Mathematics , the Man-Made Universe" by Sherman Stein (1963), "Mathematics, A Human Endeavor" by Harold Jacobs (1970), and "Excursions into Mathematics" by Beck, Bleicher and Crowe (1969). These textbooks harkened back to the Dantzig-Kasner-Newman model and defined for us how to teach that sort of Liberal Arts math course-- the tone to take and the topics to tackle. These texts asked novel questions, found mathematics in the oddest places (like in a pool hall or in an artist's studio) and treated their readers like intelligent partners in exploration. Luckily all are still available [1].

Although choice of content should not be the primary emphasis of what is now called the Excursions course, these texts and many others since agree on much of it: number theory (perfect numbers, modulo arithmetic), geometry (tiling, non-Euclidean geometries, Platonic solids, Escher etchings), Infinite sets, graph theory (map coloring, Konigsberg bridge problem), discrete math (counting, novel problem solving-- Polya heuristics), for example. With the appearance of the groundbreaking textbook "For All Practical Purposes" by the COMAP group (Freeman, 1988) came the welcome addition of such topics as fairness (voting, apportionment, fair division), management science (discrete mathematics applied to modern, real world problems), fractals, codes (bar codes, secret codes) and the notion that much mathematics is very new and relates intimately with our lives. The modern instances of Excursions courses can now chose from all these many and rich directions. There are too many strands to fit into one course, although some texts swell their pages by trying to pay homage to them all.

At Hofstra where I teach, an instructor usually chooses among two types of textbook, the (by now) traditional Dantzig-inspired approach [2] or the

COMAP-inspired one [3]. (Of the former, some have the exploratory spirit of the classics mentioned above while others are somewhat watered down, even remedial, covering topics that are nowadays found in the high school curriculum.) Some of our Hofstra teachers have found Excursions an ideal course in which to use computer software such as the Geometer's Sketchpad, LOGO, spreadsheets and BASIC. It's also a good place to experiment with students working in teams and students practicing convincing each other and the teacher of their points of view in writing and in speaking. Some few Hofstra professors have incorporated unusual themes like Mathematical Linguistics or an informal presentations of Goedel's Incompleteness Theorem and the Turing Halting Problem.

Several of the newer texts [4] are hybrids-- essentially traditional with some added COMAP-type chapters, sections or asides. Some of the older traditional texts [5] have now also added some of these extra parts, some to the point of becoming unbelievably unwieldy and some choosing a mix of topics to keep overall size manageable.

Excursions is fun to teach. Many topics may be new to the instructor or likely to be among his or her favorites. Being excited about the course you're teaching makes it much more likely that your students will enjoy it also. Nowadays when students' backgrounds are seen to be weak and innumeracy reigns, such a course may be preferable to remedial Math courses. These latter can be demoralizing for the student and so have the opposite effect than was intended, while a course filled with material that's new for the student, has obvious applicability to the student's world, and is exciting to the instructor, is more likely to be a success. And although the topics often aren't the traditional ones, still the student learns mathematical techniques (reasoning, problem solving, number properties, as well as other old mathematical wines presented in new bottles) and perhaps just as important, learns that he or she can do mathematics and that it's interesting and worthwhile. Ideally, the traditional topics get learned as by products of these exciting adventures.

Often the topics of an Excursions course are fairly independent of one another, though several smaller topics may naturally fall under one more general heading. Instead, I suggest that a good way to teach the course is to unify its content under one over-arching theme under which all the semester's many mathematical topics will fit. I've have done this using "Mathematics and Mysticism" as the theme ("Codes" and "Math in

Literature" are some other possibilities) and I now describe what I've come up with under this umbrella.

Mathematics and Mysticism

The mysterious, occult and positively spooky aspects of numbers and figures can be used as a jumping off point for more serious investigations. The mystical aspect of mathematics has been with us since history's dawning with some anthropologists even claiming that mathematics was invented for religious and not mundane practical purposes. Today we tend to dismiss this attitude as superstitious-- mathematics is a practical tool for the shopper, the carpenter; math is a reliable, useful, sometimes surprisingly powerful tool for the engineer; it is beautiful to mathematicians and philosophers, but we no longer tolerate those blatant mystics (like Pythagoras, Kepler, Buckminster Fuller) who have mistakenly gone beyond the aesthetic appreciation to an unnatural awe we moderns, by and large no longer condone.

But vestiges still do remain in academic discourse. For example, like Pythagoras who believed "all is number," we feel science will succeed in explaining the universe in mathematical terms; we also expect theories of science to be mathematically elegant otherwise they don't seem to carry the requisite explanatory punch. A theory that is not beautiful cannot be true. It is difficult also to stay detached in the face of mathematics' uncanny applicability to the physical world as it is difficult to hold the line between its aesthetic appreciation and mystical awe.

Outside academe mathematical mysticism is rampant: horoscopes, pyramid power, lucky and unlucky numbers, mystic symbols, numerology, divination. But, my friends, whatever your view, even if you abhor the mystic, don't despair-- we teachers can harness this power for good! We can use this unholy attraction to teach real mathematics and to explore valid issues of mathematical philosophy.

What is the mystical in mathematics? It's not the content but the attitude. Ahmes, the scribe of the Rhind papyrus, for example, says of his work of elementary arithmetic and geometry, "Herein are contained all the mysteries". He was in awe of what we today consider the trivial basics. In the film, "Clan of the Cave Bear," the hulking Neanderthal shaman shows the visiting CroMagnon (played by the lovely Darryl Hannah) the mystery of his miraculous counting stones. He unwraps them reverently from their animal skin cover and uses these things of magic to intone, “one, two, three, four.” The genetically superior Ms. Hannah studies the stones for a moment and then noticing that other arrangements are possible, brightly continues, “five, six, seven!” The shaman jumps back in wonder and in fear. He quickly rewraps his stones and wisely counsels, “Don’t tell the others!” Indeed, they might get burnt in the presence of such mystery.

In spite of being able to find mysticism anywhere in mathematics, some topics have a more obvious mystical reputation. Here is a short list of topics that can be woven into a Math Excursions course under a Mysticism Umbrella:

A. Geometry: Pyramidology-- does the great pyramid at Giza contain hidden knowledge? Or is Egypt’s truly greatest pyramid (according to Howard Eves) the formula for the volume of the truncated right pyramid? The Platonic solids, sacred geometry, the golden mean. Kepler, Buckminster Fuller. [6]

B. Probability, Numeration: Using chance devices for divination-- casting lots, rolling dice or astralagi, entrail reading, bone & shell cracked in the fire, fortune cookies, the I Ching, Tarot cards. Binary notation, Liebniz, nim. [7]

C. Number Theory: Numerology, gematria (the Pythagoreans, the Kabbalah), permutations. “The Incredible Dr. Matrix” of Martin Gardner. Fu Xi’s 3x3 magic square, nxn magic squares. [8,9]

Are we committing an error in honoring the mystical attitude in mathematics? --certainly much nonsense has been uttered in its name. Lest students leave the course with a skewed view, I’d recommend including a discussion on the debunking of pseudoscience and pseudomath. Good sources are Underwood Dudley, Martin Gardner, John Allen Paulos and the journal <u>The Skeptical Inquirer</u> [9]. It’s interesting to see that although these authors are active and energetic debunkers, their works show a deep fascination with matters mystical. Consider also in this regard, Gardner’s Dr. Matrix books and his debunking biography of Mary Baker Eddy.

Some years ago I was teaching a basic statistics course to adults with weak backgrounds who had been away from school for a long while. One evening as class was gathering, a student came in and passing my desk said to me nodding his head “yes” with a trance-like smile, “I did some standard deviations last night.” I thought to myself, “And...?” But there was no “and” I soon saw-- he was merely sharing with me the joy and wonder of carrying out a long computational dance with esoteric symbols. This is what the Jewish kabbalists must have felt as they went through the sequence of permutations of the name of God, and what we used to feel before we became sophisticated in our art.

We diminish our mathematical art/science/religion by cutting it off from the roots that have nourished it, still nourish it, but now secretly. I for one like to admit the attraction and suggest you can too. There needs to be a place in the Math curriculum where unusual topics like this can be explored. In Excursions the professor gets to chose umbrellas, approaches, content.
At Hofstra Math Department several years ago, we tried to get instructors of Excursions to agree on a common text, at least for one semester at a time. The effort failed and rightly so-- the charm of this course is in its freedom. This course has been successful at Hofstra. Over the years it’s grown from offering one section a semester to being tied for first place with our business-oriented course in matrices. Hooray, I say.

References:

[1] Jacobs is in its 3rd Edition with W.H.Freeman. Recently reissued are Stein by Dover Books, and Beck et al by A.K. Peters.

[2] For example: “The Nature of Mathematics,” Karl J. Smith, Brooks/Cole, “A Mathematical Sampler: Topics for the Liberal Arts,” Berlinghoff & Grant, Ardsley House and “Mathematical Ideas,” Miller et al, Addison, Wesley, Longman.

[3] Besides the important “For All Practical Purposes” mentioned above, there’s “Excursions in Modern Mathematics,” Tannenbaum & Arnold, Prentice Hall.

[4] For example, “The Heart of Mathematics: An Invitation to Effective Thinking,” Burger & Starbird, Key and “Mathematics in Life, Society and the World,” Parks et al, Prentice Hall.

[5] For example, all three texts mentioned in [2] have recently added COMAP-like sections.

[6] ”Synergetics, Explorations in the Geometry of Thinking,” and other works of Buckminster Fuller,
“The Divine Proportion,” H. E. Huntley, Dover Books,
“An Introduction to the History of Mathematics,” Howard Eves.

[7] “Games, Gods and Gambling,” F. N. David, Chaps. 2 & 3,
“I Ching,” Richard Wilhelm/C. F. Baynes.

[8] “A History of Western Philosophy,” Bertrand Russell, Chap. 3,
“Jewish Magic and Superstition,” Joshua Trachtenberg.

[9] “Numerology or, What Pythagoras has Wrought,” by Underwood Dudley, MAA Spectrum,
“Fads & Fallacies in the Name of Science,” by Martin Gardner, Dover Books,
“Innumeracy, Mathematical Illiteracy and its Consequences,”
John Allen Paulos, Vintage Books, 1988, especially Chap. 3,
The Skeptical Inquirer, CSICOP, Committee for the Scientific Investigation of Claims of the Paranormal, Paul Kurtz, Publisher.