Preface to the Robert J. Bumcrot Festschrift Papers

Robert J. Bumcrot retired from the Mathematics Department of Hofstra University after an illustrious career as researcher, educator, administrator and leader in our profession. His friends at Hofstra felt strongly that a celebration of Bob's outstanding contributions was in order and so his colleagues, co-authors and former students were invited to contribute the papers in these proceedings. These papers were presented at a Mathematics Festschrift Conference held at Hofstra University on May 11, 2001.

Bob's areas of study and research include Geometry (his original research area), Analysis/Topology, Discrete Math (especially Combinatorics & Graph Theory), Number Theory, History of Mathematics, Computer Science, Problem Solving and Mathematics Education. In the classroom he has a reputation as a demanding exponent of "tough love," and his students usually respond with stronger performances than they knew they were capable of. He has the same effect on his colleagues, especially when twice he held the office of chair, helping them achieve their higher goals to enrich their professional lives and the quality of our department and our university. He served also as chair, and then governor, of the New York Metropolitan Section of the MAA and chair or member of several local and national committees to improve the mathematics education of our nation's students.

The papers were reviewed by the editorial committee identified below. The organizers of the conference, Professors Marysia Weiss and David Knee wish to express their gratitude to Provost Herman Berliner of Hofstra University and to Dean Bernard Firestone of the Hofstra College of Liberal Arts and Sciences and his Associates for providing the funding for this conference. We also thank our colleagues in the Mathematics Department for their many-faceted support. In particular, thanks to Dean Steve Costenoble most of these articles are now available on the net at Hofprints.hofstra.edu.

Of course, invited contributors were asked to give talks in areas they felt would be of interest to Bob. As one of the speakers, Professor Harold M. Edwards of NYU pointed out, this pretty much left them with *carte blanche* because Bob's interests are so wide ranging. There are a total of 18 papers: perhaps the most represented area is Discrete Mathematics with five papers, next comes Abstract Algebra with four papers, History and Applications have three each, and Set Theory, Dynamics and Mathematics

Education with one each. We briefly describe them in that order.

Studies of techniques of Fair Division start with the assumption that the participants in the division process do not know the preferences of the other 'players.' In Raymond Greenwell's paper, "Satisfaction, Secrecy and Inequity in the Problem of Fair Division," the author takes steps in developing methods for analyzing the fair division of continuously divisible objects like cakes or pizza pies, that allow participants to expose their preferences to each other, rather than hide them.

Using the tools of Symbolic Dynamics, Sylvia Silberger's "Entropies of Tiling Systems" deals with discrete geometrical structures composed of square tiles (like 1-dim polyominoes) and the bi-infinite sequences they generate.

In "Graphs and Pigeonholes," Michael Gargano presents a sweeping array of theorems that tumble forth from the simple yet deep Pigeonhole Principle. He also shows how translating the theorems of Number Theory into the language of Graph Theory (or vice versa) can give us many new truths 'for free.'

From a pancake flipping problem first posed in the pages of the Monthly (1975), Joseph Malkevitch, in "Pancakes, Graphs and the Genome of Plants" tells a combinatorial tale that can enrich both high school and college mathematics classes and that connects with fundamental ideas in Computer Science (sorting algorithms), Genetics (how gene sequences can go awry) and Graph Theory.

The concept of a Random Graph was introduced by Erdos and Renyi in 1959. Louis V. Quintas*, Krystyna T. Balinska and Michael L. Gargano in their paper, "The Reversible Random f-Graph Process with Loops," restrict their attention to f-graphs (i.e. those graphs whose vertices are of degree not exceeding f) and then generalize the random process in several ways: 'reversibility' (where edges may either be inserted or deleted at each stage), and 'with loops' (where at each stage in the random generation of the f-graph, in addition to insertion or deletion one may try to add an inadmissible edge, thus in effect leaving the graph unchanged.) These random processes each define Transition Digraphs. The authors review the literature of results on these latter objects (mainly produced by themselves) and present new results.

Next in popularity was Abstract Algebra: Bob is an avid problem solver of challenges that appear in the Monthly and elsewhere. Hofstra colleague Mira Bhargava tells this story: In 1975, when she was a graduate student at McGill University, she submitted this problem to the Monthly: Which groups are the union of 2 of their proper subgroups? Among her problem solvers was Bob Bumcrot, soon to be welcoming her to the Hofstra Mathematics faculty. Recalling that event, Bhargava’s paper, “Groups as Unions of Subgroups”, deals with a generalization of the earlier problem, namely, When is a finite group the union of its proper (normal) subgroups? It turns out that inserting the word ‘normal’ gives a problem with an elegant answer; otherwise the classification seems to be a formidable task.

In “A Survey of Four-Dimensional C-Associative Algebras,” we are reminded that the Quarternions is the only 4-dimensional associative real division algebra. Steven Althoen* and his associates, K. D. Hansen and L. D. Kugler, survey their own work and that of others in imaginatively tinkering with the notion of associativity to produce a wealth of other 4-dimensional real algebras that, while not associative, satisfy various weakened forms of that property and display a coherent simplicity and beauty of their own.

Manjul Bhargava learned his Combinatorics from Bob when he was still a high school student, attending Hofstra to supplement his accelerated education in the early 90’s. In what was one of the high points of the Festschrift, and just one day after being awarded his Ph.D. from Princeton, Manjul electrified the audience by presenting a deep and elegant result invoking the names of Lagrange, Dickson, Ramanujan and Conway, on exactly which (positive definite) quadratic forms represent all the positive integers. His paper is titled, “The Fifteen Theorem and Generalizations.”

Although Max Dehn’s Word Problem on groups is undecidable, it and many related problem are decidable for certain restricted classes of groups (for example, Abelian groups and solvable groups). Gretchen Ostheimer*, in her paper, “Algorithms for Infinite Groups”, describes some of the practical computer-implementable algorithms she and her co-investigator, Bettina Eck have created to solve these problems for the specific inputs of the working mathematician.

Now we come to the History of Mathematics: In “On the Fundamental Theorem of Algebra,” Harold M. Edwards demands: Will the real fundamental theorem please stand up. Edward’s winning choice is not the formulation of

Gauss' Ph.D. thesis which tried to avoid the circular reasoning of the earlier attempts of Euler, Lagrange and Laplace, but rather Gauss' redo some 16 years later when he returned in style to the more symbolic approach of his predecessors, and which involves the 'algebraic' manipulation of letters in an auxiliary polynomial.

David Cassidy, advisor to the recent Broadway production of "Copenhagen" and author of an acclaimed biography of Werner Heisenberg, recaptures that exciting era in physics in his paper, "Gino Fano's Son Ugo." Ugo, who died earlier this year, was a mathematician and physicist who worked in Italy and Germany with the likes of Heisenberg and Fermi and fled to this country when, as a Jew, his position became untenable.

John Impagliazzo summarizes the history of polynomial solvability from the ancient Babylonians through to the work of Niels Abel in his expository article, "A Journey in the Mind of Galois." He then imaginatively retraces Galois' thoughts as he laid the foundations of modern group theory and illuminated its connections with the solvability of polynomials by radicals.

On to the area of Applied Mathematics: In the paper, "Spatio-temporal Variability in Ventricular Fibrillation," Harold M. Hastings* presents work by himself and mathematical and medical co-workers Steven J. Evans, Flavio H. Fenton and Alan Garfinkel, that shows how the mathematics of Differential Equations and Chaos Theory sheds light on the dynamics of the living heart. At least half a million people die of ventricular fibrillation each year and a promising approach to conquering this disease seems to be the development of drugs that can cut into the causal steps to the onset of this unstable and chaotic behavior of the beating heart. (Note: The paper included in these Proceedings differs from the one described here and delivered at the conference.)

Gregory Levine*, tells the story of Daniel Ketover, a Long Island high school student who was able to learn the basics of Quantum Mechanics on his own and well enough to collaborate with Levine in a study of the "Roots of the Majorana Polynomial and the Quantum Mechanics of Spin."

In Albert Novikoff* & David Seppala-Holtzman's paper, "Matrices, Mappings and Partial Inverses," the authors re-envision Kirchoff's Laws, abstracting them and presenting them in the language of Homological Algebra & diagram chasing.

In Set Theory, Dan Seabold reminds us of a theorem of Cantor on countable dense linear well orderings, and describes his rediscovery of its converse (originally found by Fraisse', but not well known). He then goes on to derive several consequences in, "A Converse to a Theorem of Cantor."

In their paper, "A Twisted Tensor Product on Symbolic Dynamical Systems and Ashley's Problem," Hofstra's dynamics group consisting of Hastings, Silberger, Weiss and Wu examines the equivalences of the dynamical system corresponding to the celebrated Ashley problem and the simple two-shift. It is trivial to show that the "Ashley System," A, is shift equivalent to the full two-shift, but a topological equivalence has not been established. The authors define the notion of fiber bundle via a twisted tensor product on the transition matrices. They show that topological conjugacy implies shift equivalence in this bundle context and that the "Ashley system" fits into the bundle context. This system is then compared with another system, W, where W is topologically equivalent to the full two-shift. Systems A and W have the same base space but their fiber must be constructed with a different twisting. As a result, systems A and W are shift equivalent but not bundle isomorphic.

And finally, David Knee's article is in the area of Undergraduate Education. Several of the other papers (e.g. Greenwell, Malkevitch, Levine, Gargano and Impagliazzo) can easily fit into this category as well. In "Using an Umbrella Theme in Teaching a Math Excursions Course", Knee traces the evolution of this modern college liberal arts course from Tobias Dantzig's "Number, the Language of Science" to "For All Practical Purposes" by the COMAP group. He then suggests a new way of teaching Excursions using the over-arching umbrella theme and illustrates this method using the theme, 'Mathematics and Mysticism.'

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